Open loop small-signal control-to-output transfer function of PWM buck converter for CCM: modeling and measurements

Open Loop Small–Signal Control–to–Output Transfer Function of PWM Buck Converter for CCM: Modeling and measurements

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Abstract **– A practical procedure for measuring the control–to–output transfer function of a PWM buck converter operated in continuos current mode (CCM) is given. It is shown that a simple, low as cost measurement setting can be used. The results of the presented procedure shows that the frequency behavior of a PWM buck converter highly depends on the filter capacitor equivalent series resistance (ESR). It is also shown that models of PWM buck converter presented in previous works yields accurate result only if the capacitor ESR variation with the operating frequency is taken into account. The result of this work cane be used in designing an appropriate feedback loop of the converters.**

I. INTRODUCTION

Linear small–signal models of PWM dc–dc converters operated in Continuous Current Mode (CCM) have been derived in [1] and [2] by using the state–space averaging technique. This method is based on state variables, is general, and allows for a good understanding of dynamic behavior of PWM converters. However, the inductor current ripple is neglected, moreover this method is tedious when applied to converter circuits with many parasitic components. In [3] and [4] linear averaged models of PWM dc–dc converters have been derived by replacing only the switching part of the converter with averaged circuits. A systematic method for including parasitic components into static and dynamic models of PWM converters operated at the CCM has been presented in [5].

The purpose of this paper is to give a practical procedure for measuring the control–to–output transfer function of a PWM dc–dc converter operated in CCM. Plots of the transfer function magnitude and phase as a function of frequency are drawn. These plots allow for an appropriate design of the feedback loop. Since the capacitor ESR value depends on the operating frequency and it highly affects the control–to–output transfer function, methods presented in [1]–[5] yield inaccurate results if the capacitor ESR is assumed constant and frequency independent. It is shown that by considering three values of ESR ($R_C = 2\ \Omega$ for 100 Hz $\le f <$ 900 Hz; $R_C = 1.5\ \Omega$ for 900 Hz $\le f <$ 2 kHz; $R_C = 1.3\ \Omega$ for 2 kHz $\le f \le$ 10 kHz), an accurate representation of the control–to–output transfer function is achieved by using the method proposed in [5]. Results of these papers are useful in designing the feedback loop of PWM dc–dc converters.

II. CONVERTER MODEL

The schematic circuit of a PWM dc–dc buck converter is shown in Fig. 1(a) along with the PWM modulator which varies the switch duty cycle to regulate the output voltage against load and input voltage variations. Fig. 1(b) shows the equivalent circuit of the converters including the parasitic components [5]. It has been assumed that the power MOSFET behaves as on open circuit when OFF and as a constant resistance R_{DS} when ON. The diode has been modeled as an open circuit when OFF and as dc voltage source V_F in a series resistance R_F when ON. Resistances R_L and R_C are the inductor and capacitor ESRs, respectively. Fig. 1(c) shows the dc–dc buck converter small–signal model derived by using the method given in [5]. The resistances considered in the model are R_C and

$$r = DR_{DS} + (1-D)R_F + R_L . \qquad (1)$$

The control–to–output–transfer function is [5]

$$T_P(s) \equiv \left.\frac{v_O}{d}\right|_{v_I=0} = \frac{V_{IN}R_C R}{L(R+R_C)} \frac{s+\omega_Z}{s^2 + 2\,\omega_R\,\xi\,s + \omega_R^2}$$
$$= K\frac{s+\omega_Z}{s^2 + 2\,\omega_R\,\xi\,s + \omega_R^2} \qquad (2)$$

where

$$K = \frac{V_{IN} R_C R}{L(R+R_C)} \qquad (3)$$

$$\omega_Z = \frac{1}{R_C C} \qquad (4)$$

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$$\omega_R = \sqrt{\frac{r+R}{LC(R+R_C)}} \tag{5}$$

$$\xi = \frac{C(Rr + R_C R + R_C r) + L}{2\sqrt{LC(R+r)(R+R_C)}} \tag{6}$$

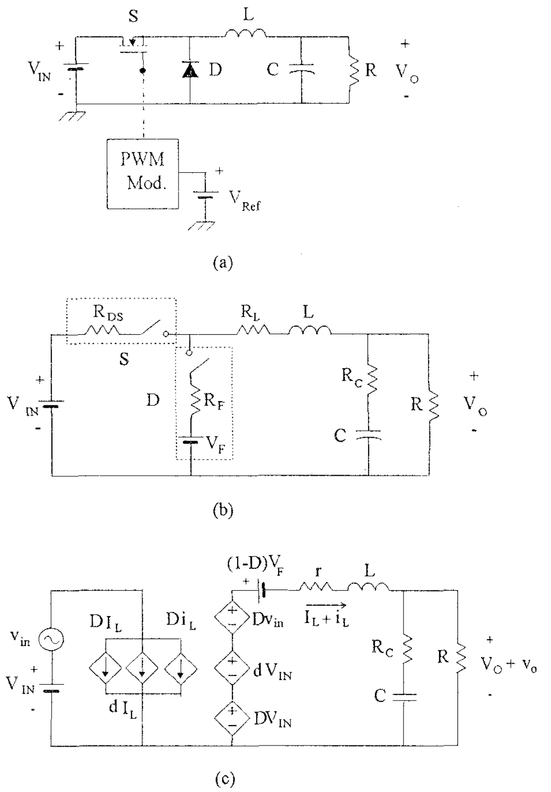

Fig. 1. PWM buck Converter [5]. (a) Schematic circuit of the ideal converter. (b) Schematic circuit including parasitic components. (c) Linear model.

III. EXPERIMENTAL VERIFICATIONS

A PWM dc–dc buck converter was assembled to meet the following design specifications
input voltage V_{IN} = 15 V;
nominal duty–cycle D = 50 %;
nominal output voltage V_O = 7.5 V;
switching frequency f_s = 93 kHz.

The schematic circuit of the buck converter tested at an open–loop operation is shown in Fig. 2.

To achieve the CCM operation of the converter, an inductance of L = 700 μH was assembled by winding 40 turns of a 40 x 0.1 Litz wire on a Siemens RM14 N27 ferrite core with an air gap of 0.3 mm. Actually, the inductor ESR increases with the operating frequency, but it does not significantly affects the control–to–output transfer function. Therefore, a constant value of $R_{L(93kHz)}$ = 300 mΩ was used in the model. This was the value of inductor ESR measured at the switching frequency. An electrolytic capacitor with a capacitance C = 47 μF was used as a filter capacitor. A Motorola MTP3N60 MOSFET with an ON resistance R_{DS} = 2.5 Ω, and an International Rectifier IR50SQ100 Schottky diode with an ON resistance R_F = 50mΩ and a threshold voltage V_F = 0.52 V were used. The values of the components shown in Fig. 2 are as follows. R = 33.6 Ω, R_1 = 7.5 kΩ; R_2 = 27 kΩ; R_3 = 10 kΩ; R_4 = 15 kΩ; R_5 = 4700 Ω; R_6 = 10 Ω; R_{ref} = 20 kΩ (trimmer); C_1 = 0.1 nF; C_2 = 2 nF; C_3 = 220 nF; C_4 = 220 nF. A PHILIPS 3F3 toroidal transformer with 20 turns wound for both the primary and the secondary winding was used to drive the MOSFET.

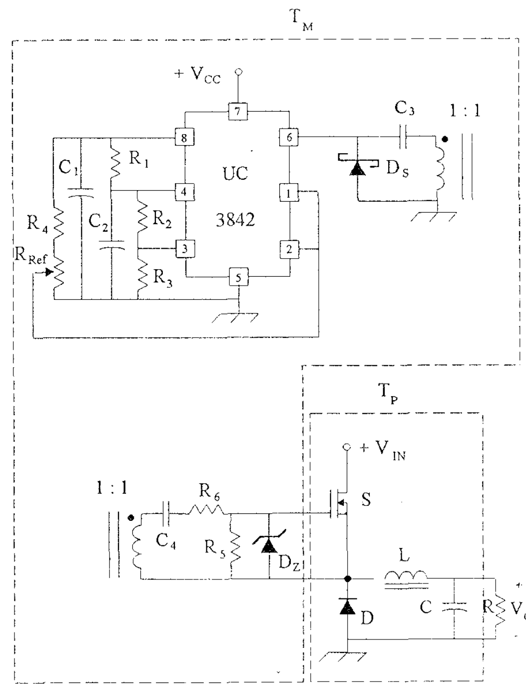

Fig. 2. Schematic of the assembled circuit. R = 33.6 Ω, R_1 = 7.5 kΩ; R_2 = 27 kΩ; R_3 = 10 kΩ; R_4 = 15 kΩ; R_5 = 4700 Ω; R_6 = 10 Ω; R_{ref} = 20 kΩ (trimmer); C_1 = 0.1 nF; C_2 = 2 nF; C_3 = 220 nF; C_4 = 220 nF; D_Z is a 12 V Zener diode; S is a Motorola MTP3N60 power MOSFET (R_{DS} = 2.5 Ω); D is an International Rectifier IR50SQ100. MOSFET driving transformer assembled by using a PHILIPS 3F3 toroidal transformer with 20 turns wound for both the primary and the secondary winding, L assembled by winding 40 turns of a 40 x 0.1 Litz wire on a Siemens RM14 N27 ferrite core with an air gap of 0.3 mm.

The plot of the measured duty cycle D against V_{ref} for the Unitrode UC3842 PWM modulator shown in Fig. 3 demonstrates that T_M is constant. Its value calculated at an operating point V_{ref} = 2,5 V is

$$T_M = \frac{\Delta D}{\Delta V_{ref}} = 1.2 = 1.58 \text{ dB}. \tag{7}$$

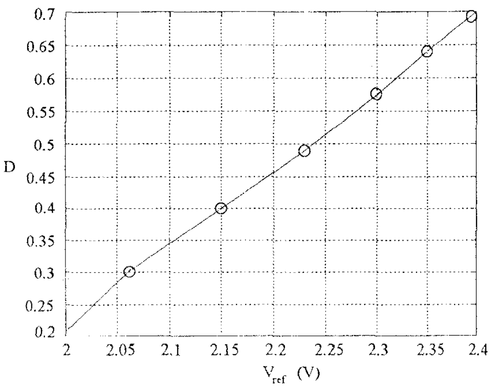

Fig. 3. Measured duty cycle D of the UC3842 controller against V_{ref}.

The capacitor ESR highly depends on the frequency and significantly affects the converter operation, three values where considered for R_C over three frequency intervals: $R_C = 1.3\ \Omega$ for 2 kHz $\leq f \leq$ 10 kHz, $R_C = 1.5\ \Omega$ for 900 Hz $\leq f <$ 2 kHz, and $R_C = 2\ \Omega$ for 100 Hz $\leq$ $f <$ 900 Hz. Fig. 4 shows the plots of T magnitude and phase for three values of capcitor ESR R_C.. These plots show that by using the low frequency value of capacitor ESR $R_C = 2\ \Omega$ over the entire frequency range the cross–over frequency is f_C = 8 kHz while using the high frequency value R_C = 1.3 Ω the cross-over frequency reduces to f_C = 6 kHz. Moreover, if the value R_C = 1.3 Ω is assumed at low frequencies the magnitute of T is 5dB higher than the value achieved by using R_C = 2 Ω. As a consequence, a constant value of R_C over the entire frequency range results in an unaccurate model.

Fig. 5 shows the test setting used for measuring the input–to–output transfer function. A sinusoidal voltage was generated by using a Philips 5192 syntetizer with the output connected to pin 2 of the UC3842 to achieve a sinusoidal variation of v_{ref} and, therefore, a dynamic duty cycle d. The amplitude of the sinusoidal voltage was 74 mV. The dc voltage of the of the reference voltage is V_{ref} = 2.25 V. As a result, the voltage perturbation was 3.3 %. For a voltage reference varying from 2.172 V to 2.328 V, the duty cycle increased from 0.45 to 0.56, that is, the duty cycle varied in the range of −10% to +10% of the nominal value D = 0.5. The voltage transfer function was measured in a frequency range 100 Hz ÷ 10 kHz. At higher frequencies the sinusoidal ac component of the output voltage had a very small amplitude and his waveform could not be shown by the scope.

A HP 54600B scope was used for measuring the time delay ΔT between v_{ref} and v_O. The phase delay was $\Phi° = 360°\ \Delta T\ f_{IN}$ where f_{IN} is the frequency of v_{ref}, and Φ is delay angle of v_O with respect to v_{ref}. Therefore, Φ gives also the phase of the control–to–output transfer function $T = T_M T_P$.

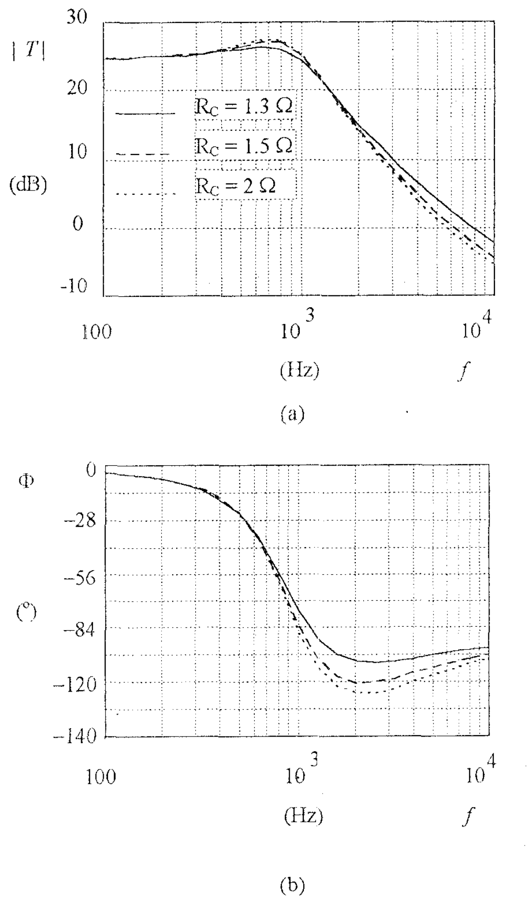

Fig. 4 Open loop control–to–output transfer function $T = T_M T_P$ for constant values of capacitor ESR R_C. (a) Magnitude (b) Phase.

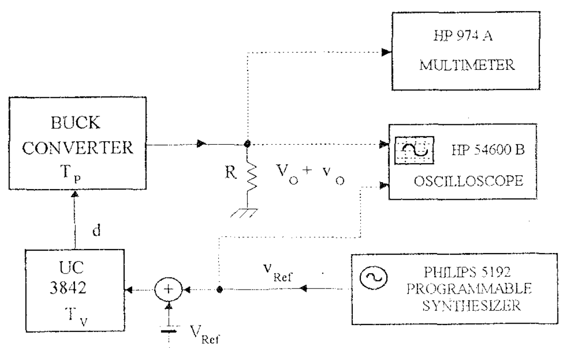

Fig. 5. The measurement setting.

Theoretical and experimental plots of the control–to–input transfer function magnitude and phase shown in Fig. 6 demonstrate that the small–signal model of Fig. 1(c) allows for an accurate representation of PWM dc–dc buck converter control–to–output transfer function for values of f up to the switching frequency, if three values of R_C in (2), (4) and (5), are considered.

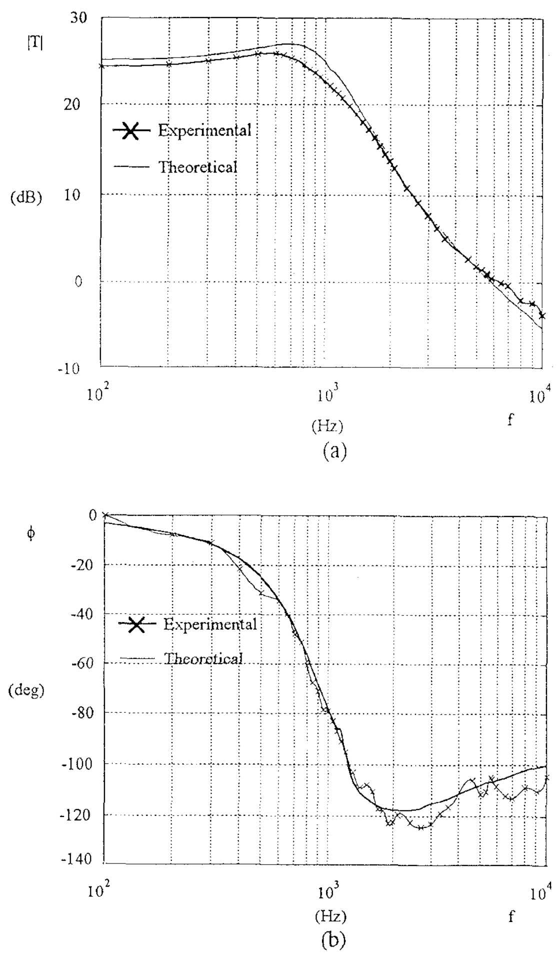

Fig. 6. Open loop control–to–output transfer function $T = T_M T_P$. (a) Magnitude (b) Phase.

IV. CONCLUSIONS

A practical procedure for measuring the control–to–output transfer function of a PWM dc–dc buck converter operated in CCM is has been presented. It has been shown that the inductance ESR does not significantly affect the frequency behavior of the converter. The buck converter operation highly depend of the filter capacitor ESR. It has been demonstrated that theoretical result given in [5] are accurate for the buck converter considered in this work if three values of the capacitor ESR are considered as follows: $R_C = 2\ \Omega$ for 100 Hz $\leq f <$ 900 Hz, $R_C = 1.5\ \Omega$ for 900 Hz $\leq f <$ 2 kHz, and $R_C = 1.3\ \Omega$ for 2 kHz $\leq f \leq$ 10 kHz. Moreover, the results shown in [5] are accurate for an duty cycle varying from −10% to +10% of its nominal value.

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